403: Algorithms and Data Structures

Analysis of Insertion Sort

Fall 2016

UAlbany

Computer Science

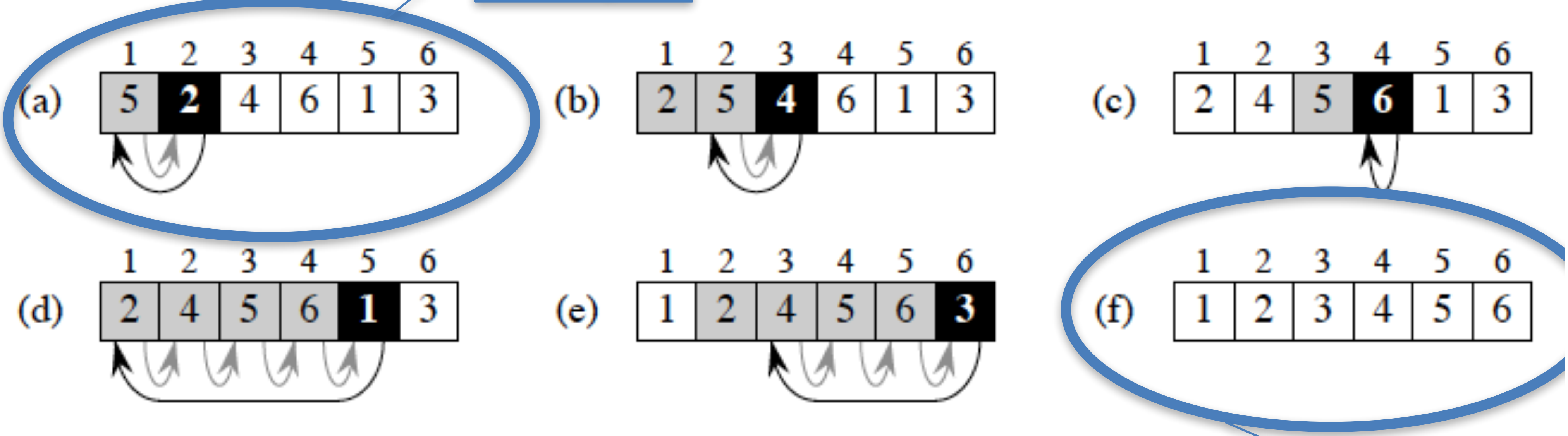
Tune-in exercise

- What is algorithm analysis?
  - Means to predict the resources needed for an algorithm execution.
- What resources are we concerned with?
  - Running time and memory
- Why do we need such resource prediction?
  - To be able to compare algorithms
  - To be able to provision resources for algorithm execution

Example of ***insertion sort*** on an instance

The algorithm

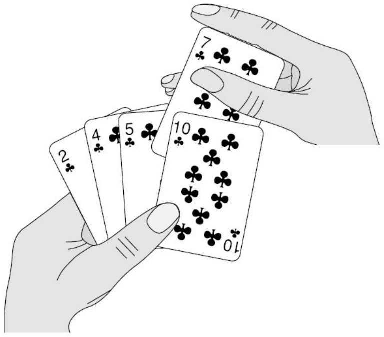
The input



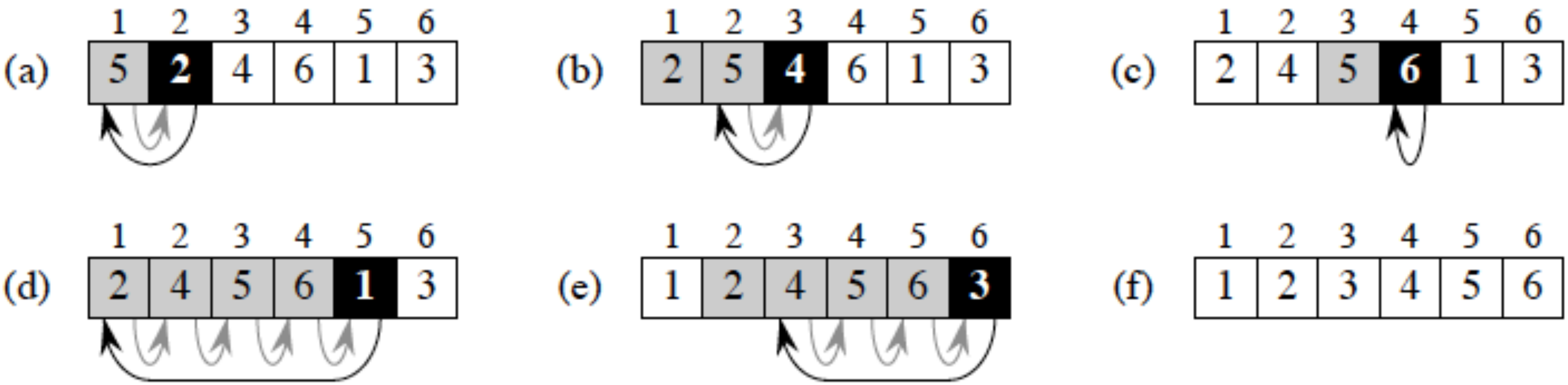
The output

- Which is the algorithm?
- Which is the input?
- Which is the output?
- What is the instance?

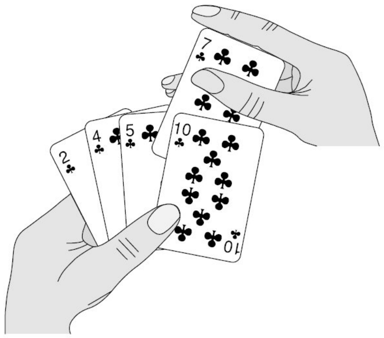
<5,2,4,6,1,3>



Example of ***insertion sort*** on an instance

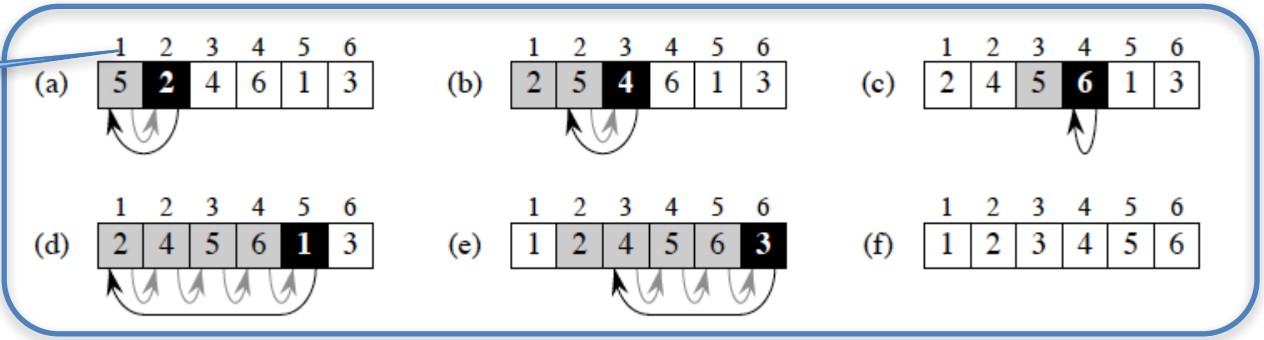


Take a minute to think on your own of what is happening at each step.



Insertion sort (pseudo code)

Input array is 1-based



```
INSERTION-SORT($A$)
for $j = 2$ to $A.length$
    $key = A[j]$
    // Insert $A[j]$ into the sorted sequence $A[1..j-1]$.
    $i = j - 1$
    while $i > 0$ and $A[i] > key$
        $A[i+1] = A[i]$
        $i = i - 1$
    $A[i+1] = key$
```

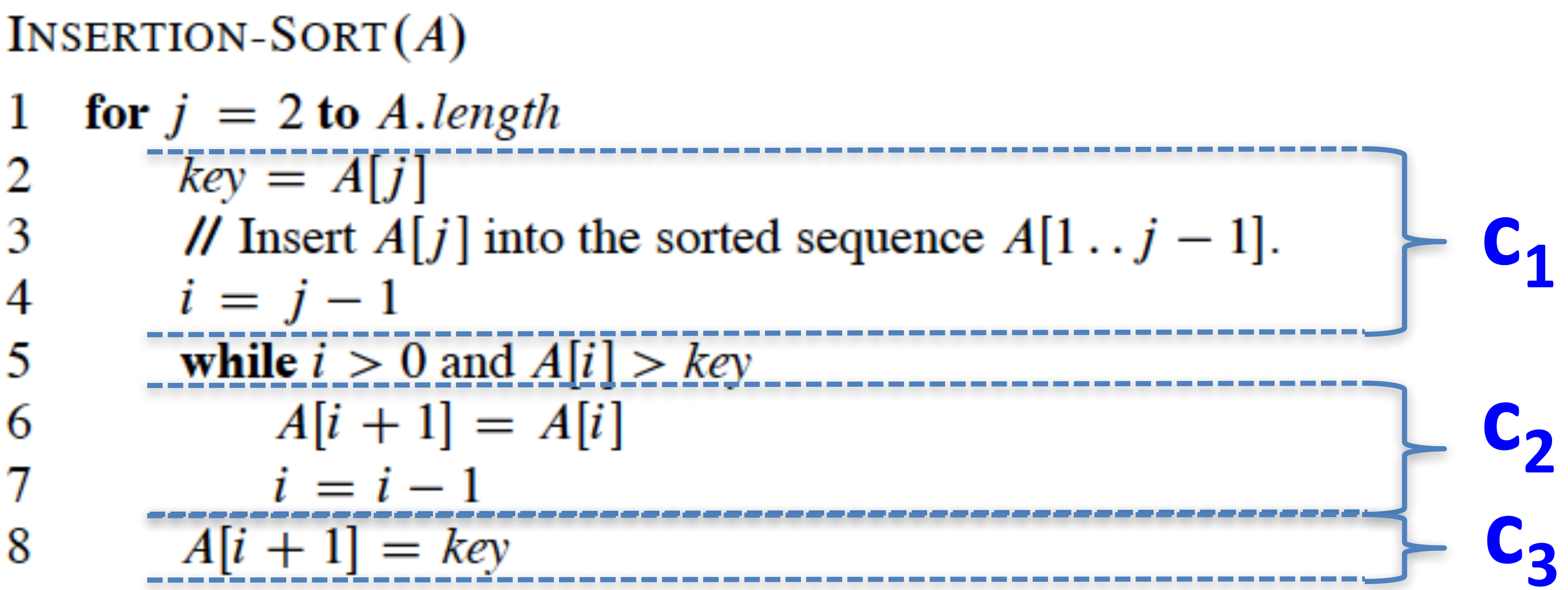
j indexes the whole array

i indexes the sorted sequence

This step can be reached when i=0 or if A[i]≤key. In both cases key is placed s.t. A[1…i] is sorted

Insertion sort -- analysis

- Recall that each primitive operation takes constant time
- Assume there are **n** numbers in the input



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INSERTION-SORT(A)
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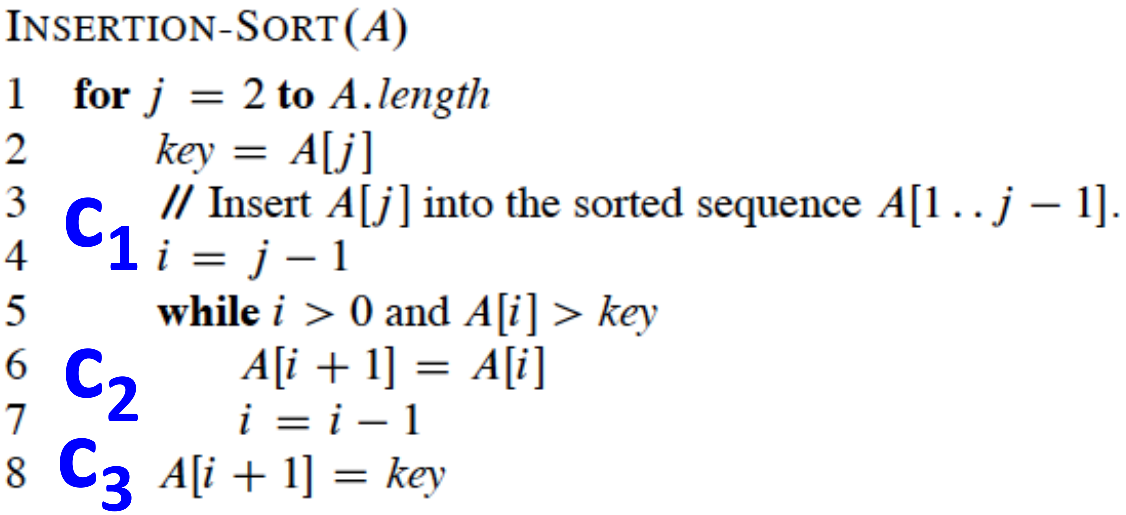
```

- c_1, c_2, and c_3 are constants and do not depend on **n**

Insertion sort -- analysis

- Assume there are **n** numbers in the input



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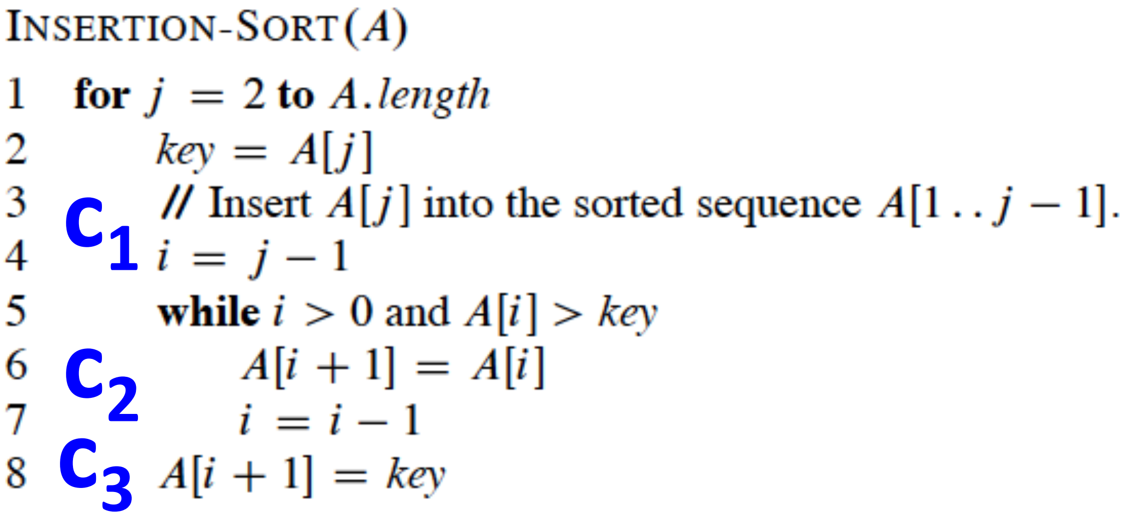
What is the time needed for the algorithm execution?

Insertion sort -- analysis

- Assume there are **n** numbers in the input



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```

- **While** loop is executed at most **j-1** times for a given **j** , so time spent in loop is at most **$(j-1)c_2$**
- Any iteration of the outer **For** loop takes at most

$$c_1 + (j-1)c_2 + c_3$$

- The overall running time of insertion sort is

$$\sum_{j=2}^{n} [c_1 + (j-1)c_2 + c_3] = d_1 n^2 + d_2 n + d_3$$

Was our analysis too pessimistic?

- We just performed a worst-case analysis of insertion sort, which gave us an *upper bound* of the running time.
- Was our analysis too pessimistic? In other words, are there instances that will cause the algorithm to run with quadratic time in n?
  - The worst-case instance is a reverse-sorted sequence $a_1, a_2, \ldots, a_n$ such that $a_1 > a_2 > \ldots > a_n$
- Since worst-case sequence exists, we say that our analysis is “tight” and ”not pessimistic”.

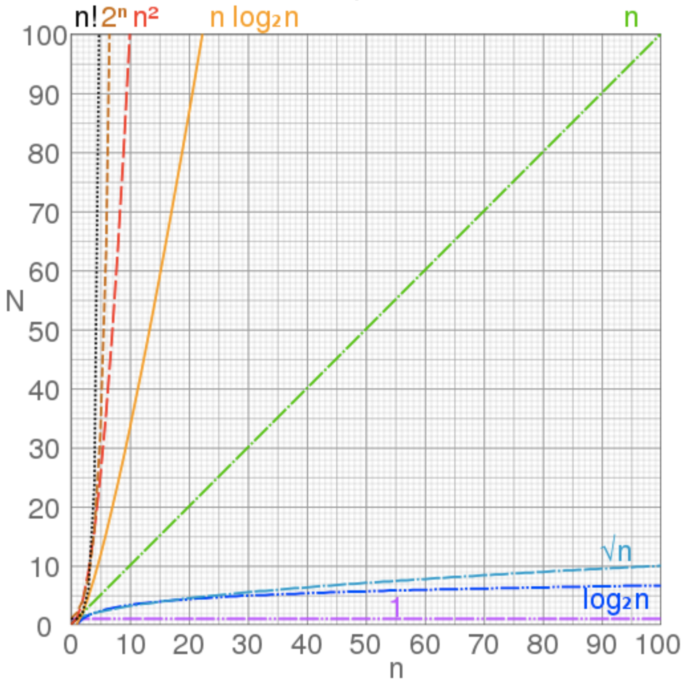
Insertion sort growth rate

- Consider insertion sort's running time as the function $d_1n^2 + d_2n + d_3$
  - The dominant part of this function is **n^2** (i.e. as n becomes large, n^2 grows much faster than n)
  - Thus, the *growth rate* of the running time is determined by the n^2 term
  - We express this as $O(n^2)$ (a.k.a. "big-oh" notation*)
  - We compare algorithms in terms of their running time

* To be formally defined later

Algorithm comparison

- Which algorithm is better?
  - We answer this question by comparing algorithms’ **O()** running times.
- Example: Compare algorithm A and B. Which one is better?
  - Algorithm A: **$O(n^2)$**
  - Algorithm B: **$O(n \log_2(n))$**



- **B is more efficient.**
- Intuitively **n^2** grows faster
- We might be wrong for small instances but when **n** is large B will be faster
- Large sizes come about very often (Facebook has 100s of millions of users)

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Announcements

- Read through Chapters 1 and 2 in the book
- Homework 1 posted, Due on Sep 7